

MAGNETIC CONTROL OF NATURAL CONVECTION IN THE HORIZONTAL BRIDGMAN CONFIGURATION: SYMMETRIC AND NON SYMMETRIC CROSS SECTIONS

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Abstract—This study deals with the electromagnetic damping of free-convective flows in cavities such as those used in the crystal growth horizontal Bridgman configuration. The cavities are filled with a dilute electrically conducting alloy and are subjected to a horizontal temperature gradient. The flow is steady and laminar under an external vertical, transversal, and uniform magnetic field. Several cross sections of the cavities were investigated and can either be centro-symmetric or not. The governing equations for such problems are two coupled partial differential equations coupled with a third integral equation for mass conservation. A finite element method has been developed and the numerical results for the variation of the velocity and the induced magnetic field in terms of the Hartmann number show a considerable decrease in convection intensity as the Hartmann number increases. Results also reveal the presence of the well known Hartmann and parallel layers. For non centro-symmetric sections, results show the way the flow reorganizes into two cells as the Hartmann number increases.

Keywords-componens: *Free convection; magnetic field; magneto-hydrodynamics; cross section; solidification; Bridgman.*

I. INTRODUCTION

The possibility of mastering the metallurgical structure, as well as the control of defects which appear during the solidification processes, allows enhancing materials properties for industrial use. Moreover, properties (mechanical, chemical and physical) depend directly upon the microstructure of materials. For instance, for several steel types, it is always possible to modify the microstructure of solidification by thermo-mechanical treatments. However, for semiconductors such an option is not possible and the microstructure of solidification acts directly on the final properties of the resultant material. Thus, the importance of the selection and the control of the microstructure are preponderant to control the properties of the material. The steered solidification processes present a privilege by allowing the control of the crystal growth during the liquid phase, and as a result the production of mono-crystal alloys of high quality.

Among steered solidification processes, the horizontal Bridgman method (HBM) may be considered for its simplicity compared to other methods and for its allowance to independently control the three experimental parameters: the temperature gradient, the concentration, and the pulling velocity. In this configuration, the single crystal develops slowly inside a melting pot of variable geometry, which is subjected to an external unidirectional axial thermal gradient and filled with an alloy fluid having an initial concentration C_0 . The cavity is pulled at a constant velocity V such that the liquid phase of the alloy in hot zone solidifies progressively in the colder one. The temperature gradient, within the liquid phase, induces free convective movements, acting on the crystalline growth by ordering the transport of doping impurities and of heat in the growth interface. The resulting natural convection, during the crystalline growth, causes non homogeneities and striations. And this is responsible for defects affecting the quality of the resultant crystals. Thus, the understanding of the movements of natural convection in the liquid phase and the ability to control them becomes a preponderant objective in an attempt to eliminate them. Since the materials are good electrical conductors in their liquid state, it is possible to use magneto-hydrodynamic (MHD) effects to modify the fluid motion and, therefore, obtain crystals which better satisfy the ever increasing requirements of technology.

Many studies had proved that magnetic fields can be used to damp the convection during the directional solidification of electrically conductive molten materials. The control of the intensity of convection by exposing the convective flow to a magnetic field of constant or variable strength and direction has been put into evidence in many works during the last three decades. For instance, the investigations of Hoshikawa *et al.* [1] and Kim *et al.* [2] were made for the control of the crystal growth in the Czochralski method. Those works have shown significant improvements of the quality of the crystals of silicon due to the reduction of temperature fluctuations responsible for the oscillating crystal growth characterized by a non uniform distribution of doping impurities in the crystal.

More recently, the experience of Campbell and Koster [3] on the crystalline growth of indium antimonide in the vertical Bridgman configuration proved that the intensity of the convection may modify the shape of the solidification front. For the horizontal configuration, Davoust *et al.* [4] had investigated numerically the MHD control of free convection. The three dimensional numerical results, obtained using a commercial numerical code (POLY3D), showed the MHD damping and the reorganization of the convective flow obtained for $0 < Ha < 100$ for a circular cell. These results also illustrated the building-up of the Hartmann layers and of the side layers when the Hartmann number increases.

The same configuration with a rectangular enclosure was also studied analytically by Garandet *et al.* [5]. In this 2D study, the authors demonstrated that for a high Hartmann number, the velocity gradient in the heart of the cavity, outside both Hartmann layers and far from walls parallel to the magnetic field, is always constant.

A numerical investigation using p-version least-squares finite element method was developed by Dennis *et al.* [6] for the prediction of the solidification under the influence of

an externally applied magnetic field. The numerical results showed considerably different flow patterns in the melt and temperature distribution in the resulting solid, in both cases of full and reduced gravities.

Mitric *et al.* [7] studied experimentally the effect of applying an alternating magnetic field on the crystal growth of gallium alloys by vertical Bridgman configuration. The results showed that by using magnetic fields of $2mT$ and $3mT$, they succeeded to maintain the radial segregation almost constant. The application of an alternating magnetic field was also experimentally studied by Diaz *et al.* [8] to investigate its effect on the decrease of radial segregation towards the end of the ingots.

Li *et al.* [9] investigated experimentally the influence of a high magnetic field on columnar dendrite growth during the directional solidification. They proved that the magnetic field changes the dendrite growth significantly. They also showed that for the MnBi dendrite, an axial high magnetic field enhanced the growth of the primary dendrite arm along the solidification direction and at a lower growth speed, a high magnetic field is capable of causing the occurrence of the columnar-to-equiaxed transition (CET). Moreover, it has also been observed [9] that a high magnetic field affects the growth of the high-order dendrite arms of the $\alpha - Al$ dendrite at a higher growth rate. The above results may be attributed to the alignment of the primary dendrite arm under a high magnetic field and the effect of a high magnetic field on crystalline anisotropy during directional solidification.

Zaidat *et al.* [10] experimentally investigated the influence of forced convection induced by a travelling magnetic field on the macro-segregation and the grain structure for Nickel alloys. They showed that this configuration can control macro-segregations and that the dendrite primary spacing may be modified by varying the applied field.

More recently, a three-dimensional numerical simulation has been performed by Jaber *et al.* [11] to study the growth of $Ge_{0.98}Si_{0.02}$ by the traveling solvent method. They applied axial and rotating magnetic fields to suppress the buoyancy convection in the melt zone. They found, by applying an axial magnetic field of various intensities (2, 10, and 22 mT), that as the axial magnetic field increases, the silicon distribution nearby the growth interface becomes more uniform. In the case of a rotating magnetic field, with different applied rotational speeds (2, 7 and 10 rpm), they found out that this kind of magnetic field has a marked effect on the silicon concentration, which changes its shape from a convex curve to a nearly flat shape as the magnetic field intensity increases.

However, one can notice that very few studies were interested in the effect of the cavity cross section shape on the velocity damping and on the induced magnetic field. To the best knowledge of the authors, the only available works are those of Garandet *et al.* [5] and Moreau *et al.* [12]. In the first reference, the authors presented an asymptotic analytical solution for the flow, in the core region, developed for the high Hartmann numbers limit. In the second reference, a numerical solution using a commercial package (FLUX-EXPERT) was presented and covered different cross section shapes.

The purpose of the present work is to develop a finite element method to study the influence of the application of a vertical magnetic field on natural convection – which

appears in the liquid phase – during the solidification of a liquid metal in the artificial crystal growth horizontal Bridgman configuration. The method had to be applied to several centro-symmetric cross section shapes (circle, square, and lozenge) and to non symmetric cross section shapes with respect to the horizontal axis (triangle and half a circle). In all cases, perfectly electrically and thermally insulating walls were considered.

The rest of the paper is organized in four sections. In the next one, the mathematical modeling of the investigated problem is presented. Essentially, the mathematical description of the problems involves two partial differential equations coupled with an integral flow conservation equation. In the third section, the numerical methodology is detailed. In brief, the weak integral form of the problem is shown and the solution strategy is discussed. In the fourth section, some of the most important results are presented and discussed for the velocity field only. Main conclusions and perspectives are drawn in the fifth and last section.

II. THE MATHEMATICAL MODEL

In this study, the geometry involves a horizontal cylinder of length L , with cross-section S . This cross section is defined by an arbitrary equation $z(y)$ that describes its contour. H is the characteristic dimension of this cross section. This cylinder is assumed to be filled with an electrically conducting liquid metal. The cylinder, which represents a typical horizontal Bridgman crucible, is submitted to a horizontal constant temperature gradient $G = \frac{\Delta T}{L}$ and to an external applied vertical uniform magnetic field \vec{B}_0 (Figure 1).

In this work, we are only interested to the fully established regime of zero flow rate driven by buoyancy. The velocity \vec{u} and the induced magnetic field \vec{b} , which is superposed to the applied magnetic field \vec{B}_0 , are then collinear and directed along the horizontal axis \vec{x} .

Under the Boussinesq approximation and in the limit of low Prandtl numbers – which is completely justified for liquid metals – the mathematical model can be obtained. It is presented in the following sub-sections.

II.1. MAGNETIC INDUCTION EQUATION

Starting from the Maxwell equations, Ohm's law and the principle of conservation of electric charge and under the classical magneto hydrodynamic assumptions [13-14], one can obtain the magnetic induction equation which can be written as:

$$\frac{\partial \vec{B}}{\partial t} - \vec{\nabla} \times (\vec{u} \times \vec{B}) + \eta \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = 0 \quad (1)$$

where \vec{u} and \vec{B} are respectively the velocity field and the total magnetic field, whereas η is the magnetic diffusivity coefficient equal to:

$$\eta = \frac{1}{\mu_m \sigma}$$

μ_m is the magnetic permeability and σ is the electric conductivity.

PREFERABLY INSERT FIG 1 ABOUT HERE

Equation (1) embeds a diffusive curl-curl second order term, a convective curl first order term, and a hyperbolic – in time – first order term. It involves the great advantage of directly relating the main hydrodynamic and electromagnetic quantities without any interference.

The respect of the magnetic field free divergence constraint, which is one of the fundamental Maxwell equations, has been widely discussed in Ben Salah's works [15-16] where the authors concluded to the necessity of the explicit respect of this condition. In the current work, and since the study is confined to the permanent fully established regime, the induced magnetic field is reduced to:

$$\vec{b} = (b, 0, 0)^t$$

The magnetic free divergence constraint implies that the only non zero component of the induced magnetic field must be independent of x :

$$b = b(y, z)$$

and the total magnetic field is then written as:

$$\vec{B} = (b(y, z), 0, B_0)^t$$

Projection of equation (1) along the x axis gives:

$$\eta \Delta b + B_0 \frac{\partial u}{\partial z} = 0 \tag{2}$$

The two other projected equations (corresponding to the y and z axes components) are useless since they do not provide any additional information.

II.2. THE NAVIER-STOKES EQUATIONS

Under the classical Boussinesq approximation, the density is written such that:

$$\rho = \rho_0(1 - \beta(T - T_0)) \tag{3}$$

and the Navier-Stokes equations describing the hydrodynamic aspects of the buoyancy driven flow of the present problem can be written as:

$$\vec{\nabla} \cdot \vec{u} = 0 \tag{4}$$

$$\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} \right) = \vec{g} - \beta(T - T_0) \vec{g} - \frac{1}{\rho_0} \vec{\nabla} p + \nu \vec{\nabla}^2 \vec{u} + \frac{1}{\rho_0 \mu_m} (\vec{\nabla} \times \vec{B}) \times \vec{B} \tag{5}$$

where ρ_0 , T , T_0 , β , ν , \vec{g} and p denote, the density at a certain reference temperature, the temperature, the reference temperature, the dilatation coefficient, the kinematic viscosity, the gravity acceleration, and the pressure, respectively. Here, it should be noted that in the limit of low Prandtl numbers, there is no need to couple equations (4) and (5) with the energy equation since the temperature distribution is controlled by conduction at the first order and may be expressed as:

$$T(x, y, z) = T_0 + Gx + \theta(y, z) \quad (6)$$

where $\theta(y, z)$ is a small correction, due to convection, to the externally applied uniform temperature gradient – which can be neglected.

In the present work, and since, as stated before, the interest is focused only on the permanent fully established regime, the velocity field can be written as:

$$\vec{u} = (u(y, z), 0, 0)^t \quad (7)$$

The projection of equation (5) on the axes of the Cartesian system of reference gives:

$$-\frac{1}{\mu_m \rho_0} B_0 \frac{\partial b}{\partial z} - \nu \Delta u + \frac{1}{\rho_0} \frac{\partial p}{\partial x} = 0 \quad (8)$$

$$\frac{1}{\mu_m \rho_0} b \frac{\partial b}{\partial y} + \frac{1}{\rho_0} \frac{\partial p}{\partial y} = 0 \quad (9)$$

$$\frac{1}{\mu_m \rho_0} b \frac{\partial b}{\partial z} + \frac{1}{\rho_0} \frac{\partial p}{\partial z} + g - \beta(T - T_0)g = 0 \quad (10)$$

It is worth noting that $(\vec{\nabla} \times \vec{b}) \times \vec{b}$ is a pure gradient which can be incorporated into the following new definition of pressure:

$$p^* = p + \rho_0 g z + \frac{b^2}{2\mu_m \rho_0} \quad (11)$$

and that equations (8)-(9) and (10) can be rewritten such that:

$$-\frac{1}{\mu_m \rho_0} B_0 \frac{\partial b}{\partial z} - \nu \Delta u + \frac{1}{\rho_0} \frac{\partial p^*}{\partial x} = 0 \quad (12)$$

$$\frac{\partial p^*}{\partial y} = 0 \quad (13)$$

$$\frac{\partial p^*}{\partial z} = \rho_0 \beta G x g \quad (14)$$

Integrating equation (14), having in mind that p^* is independent of y , yields:

$$p^* = \rho_0 \beta G x g (z - \bar{z}) \quad (15)$$

\bar{z} being an integration constant. Taking the derivative of p^* with respect to x then gives:

$$\frac{\partial p^*}{\partial x} = \rho_0 \beta G g (z - \bar{z}) \quad (16)$$

Finally, equation (12) can be reformulated as follows:

$$\nu \Delta u - \beta G g (z - \bar{z}) + \frac{1}{\mu_m \rho_0} B_0 \frac{\partial b}{\partial z} = 0 \quad (17)$$

Equations (2) and (17) represent the system of equations to be solved. These equations can be considered as two modified Poisson equations coupled through their right hand side terms. This system of equations is underdetermined since it involves three unknowns u , b , and the integration constant \bar{z} . Therefore, one needs to provide a closure relation to obtain a unique solution. Since the problem involves a nil net flow, an obvious choice for this

closure relation is the integral equation expressing the conservation of the volumetric flow over the cross section S :

$$\iint_S u \, dydz = 0 \quad (18)$$

Equation (2) is seen as the equation for the x component of the induced magnetic field b , whereas equation (17) is the equation for the x component of the velocity field. Equation (18) could be interpreted as the equation for the integration constant \bar{z} since it plays the role of filtering the adequate value for this *a priori* unknown constant.

II.3. DIMENSIONLESS FORM OF EQUATIONS AND BOUNDARY CONDITIONS

Introducing the non dimensional following variables:

$$U = \frac{uH}{v}; \quad Y = \frac{y}{H}; \quad Z = \frac{z}{H}; \quad B = \frac{b}{B_0\mu_m\sigma v}$$

one can obtain the following non dimensional form of equations:

$$\Delta B + \frac{\partial U}{\partial Z} = 0 \quad (19)$$

$$\Delta U - Gr(Z - \bar{Z}) + Ha^2 \frac{\partial B}{\partial Z} = 0 \quad (20)$$

$$\iint_{S^*} U \, dYdZ = 0 \quad (21)$$

for which S^* is the non dimensional cross section, Gr and Ha are, respectively, the Grashoff and the Hartmann numbers, defined as:

$$Gr = \frac{g\beta GH^4}{v^2}; \quad Ha = B_0H \sqrt{\frac{\sigma}{\rho\nu}}$$

The set of equations (19)-(20)-(21) constitutes the system of equations to be solved inside the dimensionless cross section which represent the domain of application of these equations. The boundary conditions are classical. On the contour of the section S , they imply the no-slip condition $u = 0$ and the continuity of b . In this work, attention is limited to the study of electrically perfect insulating walls. Then, by Ampere's law, $b = 0$. This implies that the non dimensional boundary conditions are:

$$B = 0 \quad \text{and} \quad U = 0 \quad (22)$$

III. THE NUMERICAL METHODOLOGY

III.1. SOLUTION STRATEGY

In general, the integral-partial differential set of equations (19-21) subjected to the Dirichlet boundary conditions (22) would require a rather long iterative procedure to determine the constant \bar{Z} . In the present work, this difficulty is easily accounted for because the flow rate $Q = \iint_{S^*} U \, dYdZ$ is a linear function of the constant \bar{Z} . Considering two given values \bar{Z}_1 and \bar{Z}_2 and the corresponding solutions of the partial differential equations, it is easy to show that, with the boundary conditions, $U_1 - U_2$ is an homogeneous linear function of $\bar{Z}_1 - \bar{Z}_2$, and that the flow rates can be written as:

$$Q_1 - Q_2 = \alpha(\bar{Z}_1 - \bar{Z}_2) \quad (23)$$

As a consequence, when two values of the flow rate Q_1 and Q_2 are computed, for two given values of \bar{Z}_1 and \bar{Z}_2 , constant α is readily obtained from equation (23) and the value of \bar{Z} which ensures that $Q = 0$ can then be calculated. The special case of centro-symmetric cross sections is even simpler: U is an odd function of Z , B is an even function of Z . Hence, $\bar{Z} = 0$.

In the case of non centro-symmetric cross sections, because the integral condition of zero flow rate (21) has to be satisfied, an elementary procedure, based on the fact that the flow rate Q is a linear function of the integration constant \bar{Z} , allows to circumvent the integral differential character of the problem. This procedure reads as follows:

- Solve the system of equations (19)-(20) with $\bar{Z} = \bar{Z}_1$ and compute the flow rate Q_1 with $Q_1 = \iint_{S^*} U_1 dYdZ$.
- Resolve the same system of equations with $\bar{Z} = \bar{Z}_2$ and recompute the flow rate Q_2 with $Q_2 = \iint_{S^*} U_2 dYdZ$.
- From the values of Q_1 and Q_2 , deduce \bar{Z}_3 value of \bar{Z} which ensures that $Q = 0$.
- Resolve the system (19)-(20) with $\bar{Z} = \bar{Z}_3$ for U and B .

III.2. THE VARIATIONAL FORMULATION

Let $\Omega_h = \cup \Omega_e$ be a certain partition of the non dimensional domain S^* into elements. Then, the discrete problem associated with the weak Galerkin weighted residual form of the system of equations (19)-(20) can be established as:

$$\int_{\Omega_h} \left[Ha^2 \frac{\partial B_h}{\partial Z} v_1 - \left(\frac{\partial U_h}{\partial Y} \frac{\partial v_1}{\partial Y} + \frac{\partial U_h}{\partial Z} \frac{\partial v_1}{\partial Z} \right) \right] dYdZ = \int_{\Omega_h} Gr (Z - \bar{Z}) v_1 dYdZ \quad (24)$$

$$\int_{\Omega_h} \left[- \left(\frac{\partial B_h}{\partial Y} \frac{\partial v_2}{\partial Y} + \frac{\partial B_h}{\partial Z} \frac{\partial v_2}{\partial Z} \right) + \frac{\partial U_h}{\partial Z} v_2 \right] dYdZ = 0 \quad (25)$$

where v_1 and v_2 are two test functions chosen to belong to $H_0^1(\Omega_h)$ so as to satisfy the essential conditions $v_1|_{\partial\Omega_h} = v_2|_{\partial\Omega_h} = 0$ associated with the Dirichlet boundary conditions (22). The integral formulations (24)-(25) have been interpolated on linear triangular elements. The unstructured mesh is concentrated near the walls where the Hartmann and the parallel layers are expected to develop. The deployment of the grid densities has been chosen according to the value of the Hartmann number Ha because the layers thicknesses (Ha^{-1} for Hartmann layers and $Ha^{-1/2}$ for parallel layers) diminishes with increasing Hartmann numbers.

IV. NUMERICAL RESULTS

In this section, the numerical results are presented in terms of the effect of the external magnetic field on natural convection taking place in the cavity. Results are proposed for several symmetric and non-symmetric cross-section shapes.

The first numerical test case was selected to validate or benchmark the proposed finite element code itself against an available analytical solution [17] for natural convective flow without magnetic field. The numerical solution was obtained for the case of the obvious circular cross section with a Hartmann number set to zero, $Ha = 0$ and with $Gr = 10^6$. Different grid deployment strategies were used and investigated to ensure that the solutions were grid independent. No relaxation was used in the computations. The iterations were terminated when the relative value of the residual on the velocity, from one iteration to the next, met a prescribed condition $\tilde{\epsilon}$. In the test problems considered here $\tilde{\epsilon} = 10^{-9}$ was found to be satisfactory: it yielded solutions that were essentially insensitive to further reductions in $\tilde{\epsilon}$.

Figure 2 shows the contours of the dimensionless velocity, U , in the positive quarter of the section, that is for $0 \leq Y \leq 0.5$ horizontally and $0 \leq Z \leq 0.5$, vertically. This numerical solution was compared with the analytical solution presented by Bejan et al. [17] for the fully developed natural convection flows in horizontal long pipes subject to temperature gradients for the same Grashoff number. The numerical solution is in perfect agreement with the analytical one and therefore cannot be distinguished in Figure 2.

INSERT FIG 2

After preliminary verification, the interest was focused on the variation of the cross-section shape of the crucible. The investigated shapes are the following: circle, square, lozenge, triangle, and semi-circle (Fig. 3). For the whole set of results presented herein, the Grashoff number is set to $Gr = 10^5$, unless differently specified.

INSERT FIG 3

IV.1. CENTRO-SYMMETRIC CROSS SECTIONS

Since equations (19) and (20) are linear and symmetric with respect to Z and anti symmetric with respect to Y and that the investigated cross sections are symmetric with respect to both axes, the solutions for U and B are symmetric with respect to Z and anti-symmetric with respect to Y . Concretely, this indicates that the solution can be obtained for a quarter of the domain only (the hatched region in Fig. 3(a). Figures (4) to (6) show the contours of the non dimensional velocity, U , for two values of Hartmann number, Ha , in circular, square, and lozenge cross sections, respectively.

INSERT FIG 4

INSERT FIG 5

INSERT FIG 6

In all figures (4-6), one can notice the Lorentz electromagnetic braking or damping effect of the convective flow. For instance, in Fig.6(a), where the Hartmann number is equal to 50, the maximum value of the dimensionless velocity is -45 , whereas in

Fig.6(b), for which the Hartmann number is equal to 100, this maximum value reaches -18 , only. This clearly indicates that the magnetic field slows down convection and should produce crystals with fewer defects. Figures 4 and 5 show the same behavior for the dimensionless velocity as maxima are dropping from -30 to -9 and from -55 to -28 , respectively. Hence, for each cross-section, it is obvious that the numerical values of the dimensionless velocity show a considerable (and expected) damping of convection as the Hartmann number increases. Other Hartmann numbers were also investigated but results are not presented: similar trends were observed..

Moreover, the three figures clearly illustrate the presence of the Hartmann layers. These layers develop along the solid boundaries normal to the external applied magnetic field direction. In figures 4 and 6, these layers develop at the top of the sections, close to $Y=0$ and $Z=0.5$ whereas in figure 5, it is along the upper boundary ($0 \leq Y \leq 0.5$ and $Z=0.5$) that the Hartmann boundary is depicted. In these layers, which are necessary for the respect of the non slip condition, the essential of the velocity gradients and of the viscous forces is concentrated. Outside these layers, the viscous forces become less important and the flow is mainly organized due to equilibrium between convective and electromagnetic forces. As the Hartmann number increases, it can be seen that the layers are more and more concentrated in the immediate vicinity of the walls.

Now, if the results were to be presented using the same scale for velocity for both $Ha = 50$ and $Ha = 100$, one would immediately see that for the highest value of Ha , the velocity would be very small in a large part of the domain (not shown here). Here, the results are presented for different velocity scales to clearly present the isocontours. This can partly be observed in the case of lozenge sections (figure 6) in the central region for high numbers of Ha . No contours are drawn in the central region of the cross section in figure 6, indicating that the velocity is almost zero there and that a total blocking of convection occurs.

Another aspect that should be underlined concerns the square cross section (figure 5). Along the walls vertical wall ($Y=0.5$), the parallel layers develop, the center of the circular cell is shifted towards both boundaries, as the Hartmann number increases.

The above mentioned results (figure 4-6) clearly indicate that a strong magnetic field could be very interesting for the control of crystal growth by horizontal Bridgman methods.

Figures (7) and (8), which concern the cavity involving a square cross section only, present the variation of the dimensionless velocity defined such as $U \frac{Ha^2}{Gr}$ as a function of the width Y for the location where $Z = 0.25$ and the variation of the same dimensionless velocity in terms of the height Z at $Y = 0$ (the vertical symmetry plane aligned with gravity and the magnetic field), respectively. The proposed curves can somewhat be considered as slices of the previous figures (4-6) along the vertical axis and a line that crosses the quarter of the domain at mid-height.

In figures (7) and (8), the profiles of the velocity clearly show the Hartmann and the parallel layers development, respectively. In Figure 7, one can immediately notice that when Ha becomes important ($Ha=100$, red curve), the velocity gradient at the vertical wall (at $Z = 0.25$) is very high, that is the Hartmann layer is very thin. In this case, the parallel layer exhibits an “over” velocity, a maximum velocity at about $Y=0.44$, which is a typical signature of such layers.

In Figure 8, in the core region, the dimensionless velocity $U \frac{Ha^2}{Gr}$ is independent of Z at $Y = 0$ and varies linearly in terms of Z . The components of the velocity gradient in the core region could be deduced as:

$$\frac{\partial U_c}{\partial Z} = -\frac{Gr}{Ha^2}; \quad \frac{\partial U_c}{\partial Y} = 0$$

This result is in agreement with an analytical behavior predicted in [5]. This result reinforces the validity of the proposed FEM.

Figure 9, which concerns the cavity with a circular cross section, presents the variation of the dimensionless velocity $U \frac{Ha^2}{Gr}$ in terms of the height Z at $Y = 0$. Again, the building up of the Hartmann layer is clearly shown as the Hartmann number increases and the dimensionless velocity rate of change in terms of Z can be recomputed to be $-\frac{Gr}{Ha^2}$. And to the best of our knowledge, this is the first time such a result is presented.

INSERT FIG 7

INSERT FIG 8

INSERT FIG 9

IV.2. NON CENTRO SYMMETRIC CROSS SECTIONS

For the reasons similar to those listed in section IV.1, the solutions U and B are symmetric with respect to Z and the solution can be obtained only for a half of the domain (figure 3). Figure (10) shows the contours of the non dimensional velocity field for two values of the Hartmann number Ha in the case of the cavity with a triangular cross section.

INSERT FIG 10

Again here the electromagnetic damping of convection is obvious. The maximum value of the non dimensional velocity in the section decreases by a factor 2 from - 200 when $Ha=50$ to - 100 when $Ha = 100$. One can also observe the building up of the Hartmann layer along the horizontal wall (near $Z = 0$, about $Y = 0.3$) and along the top of the oblique wall (near $Y = 0$). It is also noticeable that the flow involves two distinct directions; the velocity is negative in a part of the section and positive in the rest of the section. In Figure

10, this delimitation for which $U = 0$ occurs between the orange and yellow contours. As Ha increases, the orientation of the delimitation between the two zones is more and more vertical, that is the slope of the zero velocity contour is steeper for $Ha=100$ than for $Ha=50$.

Figure (11) shows the contours of the non dimensional velocity field for $Ha=5$ and $Ha=100$ in the case of the cavity with a semi circular cross section. A low value of Ha was chosen to observe whether or not the flow reversal would occur along a more horizontal line for low Ha . Figure 11(a) indicates that this is the case as the two cells are almost superposed in this case. Here one should note that the maximum velocity drops by a factor 12 when Ha increase from 5 to 100.

INSERT FIG 11

V. CONCLUSIONS

In this work, a finite element method implemented on an unstructured triangular linear mesh for the solution of the magneto hydrodynamic problem associated with the study of the electromagnetic damping of the natural convection in the crystal growth horizontal Bridgman configuration is presented. The model describing the fluid phase of the process is composed of two partial differential equations of Poisson type coupled through their right hand side terms and a third integral expressing the conservation of the volumetric flow.

The resulting finite element method code was validated by use of an analytical solution for the case of fully developed natural convection flows in straight horizontal long pipes. Numerical results have then been obtained for five cross section shapes: circular, square lozeng, triangle and a half a circle, in terms of the velocity and of the induced magnetic field (not shown) distribution. For a given Grashoff number Gr , the Hartmann number Ha has been systematically changed to put into evidence the electromagnetic damping and the associated building of the Hartmann and the parallel layers. The over velocities, which are the signature of the parallel layers, have been underlined in the case of square cross sections. In the case of non centro-symmetric cross sections, the results show how the flow is reorganized in order to respect the conservation of the volumetric flow within the domain. The numerical results have been compared, in two cases, with available analytical results. In one case, it was obtained in the asymptotic limit of very high Hartmann numbers Ha and demonstrated that the velocity $U \frac{Ha^2}{Gr}$ varies linearly with position. This was found in excellent agreement with the reference solution.

The natural continuation of this numerical study is to extend the investigations to the x-y plane in order to understand what really happens within the recirculation layers, since one of these layers is necessarily in the vicinity of the solidification front. The mathematical model equations are non linear and thus present a high challenge especially because the non linearity is essentially confined to a very thin layer whereas this non linearity is almost negligible in the rest of the plane.

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